

Composite Functions

Another way of combining functions is to use the output of one function as the input of another. This is called the *composite function*, $f \circ g$. It is defined as:



To evaluate $f \circ g$, we must first apply the function g to x . Then, apply the function f to the result.

Example 1: If $f(x) = \sqrt{x}$ and $g(x) = x + 5$, find the following:

a) $f(36)$

b) $g(-4)$

c) $f[g(20)]$

d) $(g \circ f)(16)$

e) $f(g(x))$

f) $g(f(x))$

Note: $f(g(x)) \neq g(f(x))$
 $(f \circ g)(x) \neq (g \circ f)(x)$

g) $(f \circ f)(81)$

Example 2: Given $f(x) = \frac{1}{x}$ and $g(x) = x^2 - 4$

a) State the domain and range of f

b) State the domain and range of g

c) Find $(f \circ g)(x)$ and state the domain

d) Find $g(f(x))$ and state the domain

Example 3: Given $f(x) = \sin x$ and $g(x) = \frac{1}{x}$

a) State the domain of f and of g

b) Find $(f \circ g)(x)$ and state the domain

c) Find $(g \circ f)(x)$ and state the domain

Writing a Function as a Composite

In Calculus, it is often necessary to write a function as the composition of two simpler functions.

Example 4: For the given function y , determine functions f and g such that $y = f(g(x))$

a) $y = \sqrt{3x^3 - 7}$

b) $y = \frac{3}{2x^2 + 1}$

c) $y = (2x^3 + 5x^2 + 7)^5$